



education

Department of
Education
FREE STATE PROVINCE

**PREPARATORY EXAMINATION/
VOORBEREIDENDE EKSAMEN**

GRADE/GRAAD 12

**MATHEMATICS P2/
WISKUNDE V2**

SEPTEMBER 2020

MARKS/PUNTE: 150

**MARKING GUIDELINES/
NASIENRIGLYNE**

**This marking guidelines consists of 13 pages./
Hierdie nasienriglyne bestaan uit 13 bladsye.**

NOTE:

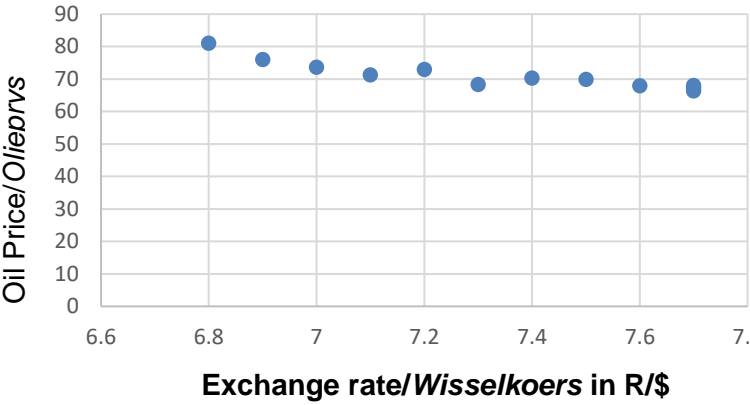
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

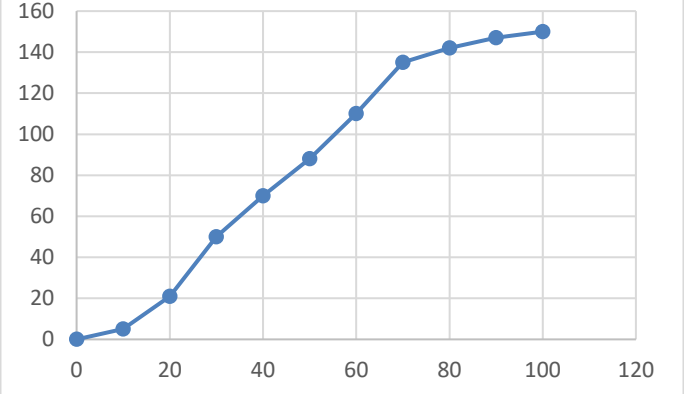
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY/MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN die rede beide korrek is</i>

QUESTION/VRAAG 1

1.1	<p style="text-align: center;">Scatter Plot/Spreidiagram</p>  <p style="text-align: center;">Exchange rate/Wisselkoers in R/\$</p>	<p>✓ 4 points correctly plotted ✓ 9 points correctly plotted ✓ All points</p> <p style="text-align: right;">(3)</p>
1.2	$y = 158,67 - 11,96x$	<p>✓✓✓ Correct equation</p> <p style="text-align: right;">(3)</p>
1.3	$r = -0,91$	<p>✓ value of r</p> <p style="text-align: right;">(1)</p>
1.4	<p>Exchange rate increase, oil price decrease/<i>Wisselkoers verhoog, olieprys verlaag</i> OR/OF Strong Negative correlation/<i>Sterk negatiewe korrelasie</i></p>	<p>✓ ✓ reason</p> <p style="text-align: right;">(2)</p>
1.5	$y = 71,05$	<p>✓ $y = 71,05$</p> <p style="text-align: right;">(1)</p>
1.6	Standard deviation/ <i>Standaard afwyking</i> : $\sigma = 4,09$	<p>✓ $\sigma = 4,09$ (1)</p>
1.7	$71,05 + 2(4,09) = 79,23$ (With calculator 79,22) December	<p>✓ 79,23 ✓ December/<i>Desember</i></p> <p style="text-align: right;">(2)</p>
		[13]

QUESTION/VRAAG 2

2.1	<p style="text-align: center;">OGIVE/OGIEF</p> 	<p>✓grouded ✓Cf ✓upper limit ✓curve</p> <p style="text-align: right;">(4)</p>
2.2.1	<p>(8;144) 144 below 85 % (accept/aanvaar 144-146)</p>	<p>✓(8;144) ✓144 (2)</p>
2.2.2	<p>$Q_2 = 42,77$ (accept/aanvaar 41-43)</p>	<p>✓$Q_2 = 42,77$</p> <p style="text-align: right;">(1)</p>
		[7]

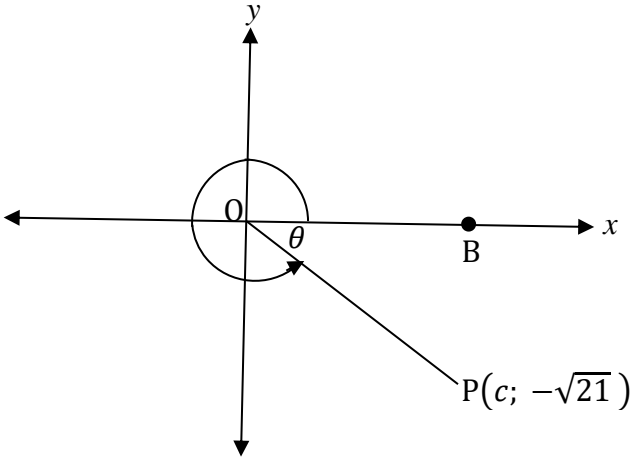
QUESTION/VRAAG 3

3.1	$M_{AD} = \frac{7-(-3)}{1-(-4)}$ $= 2$	✓ Substitution into the correct formula ✓ Answer (2)
3.2	AD// BC $M_{AD} = M_{BC} = 2$ $-8 = 2(2) + c$ $\therefore y = 2x - 4$	✓ $M_{BC} = 2$ ✓ sub ✓ Answer (3)
3.3	At F $y = 0$ $0 = 2x - 4$ $\therefore x = 2$ $\therefore F(2; 0)$	✓ $0 = 2x - 4$ ✓ $x = 2$ (2)
3.4	$B(x; y) - 7 C(x + 2; y - 5)$ $A(1; 7) \rightarrow B'(3; 2)$ OR $x_B = -2 + (1 + 4) = 3$ $y_B = -8 + (7 + 3) = 2$	✓ ✓ $B'(3; 2)$ (2)
3.5	$M_{BC} = \tan \theta = 2$ $\theta = 63,43^\circ$ $M_{DC} = \frac{-8 - (-3)}{-2 - (-4)} = \frac{-5}{2}$ $\tan \beta = \frac{-5}{2}$ $\beta = 180^\circ - 68,20^\circ = 111,80^\circ$ $\alpha = 111,80^\circ - 63,43^\circ = 48,37^\circ$	✓ $\theta = 63,43^\circ$ ✓ $\frac{-5}{2}$ ✓ $\beta = 111,80^\circ$ ✓ $\alpha = 48,37$ (4)
3.6	$DC = \sqrt{(-4 + 2)^2 + (-3 + 8)^2}$ $= \sqrt{29}$ $CF = \sqrt{(-2 - 2)^2 + (-8 - 0)^2}$ $= \sqrt{80}$ $\text{Area/Oppvl } \Delta DCF = \frac{1}{2} DC \cdot CF \sin \alpha$ $= \frac{1}{2} \sqrt{29} \cdot \sqrt{80} \sin 48,37^\circ$ $= 18 \text{ units/eenhede}$	✓ sub correct formula ✓ $\sqrt{29}$ ✓ sub correct formula ✓ $\sqrt{80}$ ✓ sub correct formula ✓ 18 units (6)
		[19]

QUESTION/VRAAG 4

4.1	$x^2 + y^2 + 2x + 6y + 2 = 0$ $x^2 + 2x + 1 + y^2 + 6y + 9 = -2 + 10$ $(x + 1)^2 + (y + 3)^2 = 8$ $M(-1; -3)$	$\checkmark 8$ $\checkmark\checkmark M(-1; -3)$ (3)
4.2	$r = \sqrt{8}$	$\checkmark r = \sqrt{8}$ (1)
4.3	$x^2 + (x - 2)^2 + 2x + 6(x - 2) + 2 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x + 3 = 0$ $(x + 3)(x - 1) = 0$ $x = -3$ or $x = 1$ $y = -3 - 2 = -5$ $\therefore D(-3; -5)$	\checkmark substitution $\checkmark x^2 + 2x + 3 = 0$ $\checkmark (x + 3)(x - 1) = 0$ $\checkmark\checkmark D(-3; -5)$ (5)
4.4	Angle between radius/diameter and tangent/ <i>Hoek tussen radius/deursnee en raaklyn</i>	$\checkmark R$ (1)
4.5	$MB^2 = MD^2 + DB^2$ Pyth $= (\sqrt{8})^2 + 4(\sqrt{2})^2$ $= 40$ $MB = \sqrt{40}$ radius of Circle/radius van sirkel C_2	$\checkmark S$ $\checkmark (\sqrt{8})^2 + 4(\sqrt{2})^2$ $\checkmark 40$ $\checkmark MB = \sqrt{40}$ (4)
4.6	$(x + 1)^2 + (y + 3)^2 = 40$	\checkmark Equation of C_2 (1)
4.7	Distance from/Afstand van $(2\sqrt{5}; 0)$ to centre/tot middelpunt $= \sqrt{(2\sqrt{5} + 1)^2 + (0 + 3)^2}$ $= 6,24$ $6,24 < 6,32 (\sqrt{40})$ Distance to centre < radius of circle/Afstand tot middelpunt < radius van die sirkel $\therefore (2\sqrt{5}; 0)$ lies inside/lê binne	\checkmark correct sub $\checkmark 6,24$ $\checkmark 6,24 < 6,32 (\sqrt{40})$ \checkmark lies inside (4)
4.8	$M_{MDP} = \frac{-3 - (-5)}{-1 - (-3)} = \frac{2}{2} = 1$ $\therefore M$ tangent = -1	$\checkmark M_{MDP} = 1$ $\checkmark M$ tangent = -1 (2)
		[21]

QUESTION/VRAAG 5

<p>5.1.1</p>	 <p> $c^2 + (\sqrt{21})^2 = 5^2$ $c^2 = 25 - 21$ $c^2 = 4$ $\therefore c = 2$ </p>	<p>✓ Subst. into pyth</p> <p>✓ Answer (2)</p>
<p>5.1.2 a)</p>	<p>$\cos \theta = \frac{2}{5}$</p>	<p>✓ Answer (1)</p>
<p>b)</p>	<p> $\frac{-\sqrt{21}}{2} + \left(\frac{-\sqrt{21}}{5}\right)^2$ $-5\sqrt{21} + 42/50$ </p>	<p>✓ $\frac{-\sqrt{21}}{2} + \left(\frac{-\sqrt{21}}{5}\right)^2$</p> <p>✓ Answer (2)</p>
<p>c)</p>	<p> $2 \sin \theta \cos \theta$ $= 2 \left(\frac{-\sqrt{21}}{5}\right) \left(\frac{2}{5}\right)$ $= \frac{-4\sqrt{21}}{25}$ </p>	<p>✓ $2 \left(\frac{-\sqrt{21}}{5}\right) \left(\frac{2}{5}\right)$</p> <p>✓ Answer (2)</p>
<p>5.2</p>	<p> $\frac{(-\sin x) \cdot \tan x \cdot \cos(360^\circ - 30^\circ)}{(-\sin x)^2}$ $= \frac{-\sin x \cdot \tan x (\cos 30^\circ)}{\sin^2 x}$ $= \frac{-\sqrt{3}}{2} \tan x \div \sin x$ $= \frac{-\sqrt{3}}{2 \cos x}$ </p>	<p>✓ $-\sin x$</p> <p>✓ $(-\sin x)^2$</p> <p>✓ $\frac{-\sqrt{3}}{2}$</p> <p>✓ ✓ $\frac{-\sqrt{3}}{2 \cos x}$</p> <p>(5)</p>
		<p>[12]</p>

QUESTION/VRAAG 6

6.1	$LHS = \frac{2 \sin^2 x}{2 \frac{\sin x}{\cos x} - 2 \sin x \cos x}$ $= \frac{2 \sin^2 x}{2 \sin x \left(\frac{1}{\cos x} - \cos x \right)}$ $= \frac{\sin x}{1 - \cos^2 x} \quad \therefore LHS = RHS$ $\frac{\sin x}{\cos x}$ $= \sin x \times \frac{\cos x}{\sin^2 x}$ $= \frac{\cos x}{\sin x}$	<p>✓ $\frac{\sin x}{\cos x}$</p> <p>✓ $2 \sin x \cos x$</p> <p>✓ $2 \sin x \left(\frac{1}{\cos x} - \cos x \right)$</p> <p>✓ $\frac{\sin x}{1 - \cos^2 x}$</p> <p>✓ $\sin^2 x$</p> <p>(5)</p>
6.2	$\sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ$ $= \sin^2 20^\circ + [\sin(60^\circ - 20^\circ)]^2 + [\sin(60^\circ + 20^\circ)]^2$ $= \sin^2 20^\circ + [\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ]^2$ $+ [\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ]^2$ $= \sin^2 20^\circ + \left[\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ \right]^2$ $+ \left[\frac{\sqrt{3}}{2} \cos 20^\circ + \frac{1}{2} \sin 20^\circ \right]^2$ $= \frac{3}{2} \sin^2 20^\circ + \frac{3}{2} \cos^2 20^\circ$ $= \frac{3}{2} (\sin^2 20^\circ + \cos^2 20^\circ)$ $= \frac{3}{2} (1)$ $= \frac{3}{2}$	<p>✓✓ compound</p> <p>✓ sub of special angle</p> <p>✓ simplification</p> <p>✓ common factor</p> <p>✓ square identity</p> <p>✓ answer</p> <p>(7)</p>
[12]		

QUESTION/VRAAG 7

7.1	<p>In ΔABC</p> $\sin\theta = \frac{h}{AC}$ $\therefore AC = \frac{h}{\sin\theta}$	<p>✓ trig ratio</p> <p>✓ answer (2)</p>
7.2	<p>In ΔABD</p> $\sin\alpha = \frac{h}{AD}$ $\therefore AD = \frac{h}{\sin\alpha}$	<p>✓ trig ratio</p> <p>✓ answer (2)</p>
7.3	$CD^2 = AC^2 + AD^2 - 2AC \cdot AD \cos\beta$ $= \left(\frac{h}{\sin\theta}\right)^2 + \left(\frac{h}{\sin\alpha}\right)^2 - 2 \cdot \frac{h}{\sin\theta} \cdot \frac{h}{\sin\alpha} \cos\beta$ $= \frac{h^2}{\sin^2\theta} + \frac{h^2}{\sin^2\alpha} - \frac{2h^2 \cos\beta}{\sin\theta \sin\alpha}$	<p>✓ cosine rule</p> <p>✓ substitution</p> <p>✓ answer (3)</p>
		[7]

QUESTION/VRAAG 8

8.1	<p>$\cos\frac{x}{2} = \sin(x - 30^\circ)$</p> $= \cos[(90^\circ - (x - 30^\circ))]$ $= \cos(120^\circ - x)$ $\therefore \frac{x}{2} = 120^\circ - x + 360^\circ \cdot k$ $x = 240^\circ - 2x + 720^\circ \cdot k$ $x = 80^\circ + 240^\circ \cdot k$ <p style="text-align: center;">OR</p> $\frac{x}{2} = 360^\circ - (120^\circ - x) + 360^\circ \cdot k$ $\frac{x}{2} = 360^\circ - 120^\circ + x + 360^\circ \cdot k$ $\frac{x}{2} = 240^\circ + x + 360^\circ \cdot k$ $x = 480^\circ + 2x + 720^\circ \cdot k$ $-x = 480^\circ + 720^\circ \cdot k$ $\therefore x = -480^\circ - 720^\circ \cdot k \text{ where/waar } k \in \mathbb{Z}$ $x_A = -160^\circ \text{ and/en } x_B = 80^\circ$	<p>✓ $\cos(120^\circ - x)$</p> <p>✓ $80^\circ + 240^\circ \cdot k$</p> <p>✓ $240^\circ + x + 360^\circ \cdot k$</p> <p>✓ $-480^\circ - 720^\circ \cdot k$</p> <p>✓ $x_A = -160^\circ$</p> <p>✓ $x_B = 80^\circ$</p> <p style="text-align: right;">(6)</p>
8.2	$-150^\circ < x < 30^\circ$	<p>✓ ✓ Critical values</p> <p>✓ Notation</p> <p style="text-align: right;">(3)</p>
		[9]

QUESTION/VRAAG 9

9.1	$AB^2 = AO^2 + BO^2 - 2(AO)(BO) \cos \hat{O}_1$ $\left(\frac{3}{2}x\right)^2 = x^2 + x^2 - 2x^2 \cos \hat{O}_1$ $\frac{9}{4}x^2 - 2x^2 = -2x^2 \cos \hat{O}_1$ $\cos \hat{O}_1 = \frac{1}{4}x^2 \div 2x^2$ $= \frac{1}{8}$ $\therefore \hat{O}_1 = 97^\circ$	<p>✓ cosine rule</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ $\frac{1}{8}$</p> <p>✓ $\hat{O}_1 = 97^\circ$</p> <p>(5)</p>
9.2	<p>$\hat{C} = 48,5^\circ$ angle at centre/<i>hoek by middelpunt</i> = twice angle at circ/<i>twee keer hoek by sirk</i></p> <p>$\hat{B}_1 + \hat{B}_2 = 83^\circ$ int angles of/<i>binne hoeke van Δ</i></p> $\frac{AC}{\sin 83^\circ} = \frac{15}{\sin 48,5^\circ}$ <p>$\therefore AC = 19,88$</p>	<p>✓ S</p> <p>✓ S</p> <p>✓ sine rule</p> <p>✓ $AC = 19,88$</p> <p>(4)</p>
9.3.1	<p>$\hat{Q}_1 = 32^\circ$ tan chord theorem/<i>tan koordstelling</i></p>	<p>✓ ✓ S/R</p> <p>(2)</p>
9.3.2	<p>$\hat{Q}_1 = \hat{P}_4 = 32^\circ$ angle opp = sides/<i>hoeke teenoor = sye</i></p>	<p>✓ ✓ S/R</p> <p>(2)</p>
9.3.3	<p>$\hat{M}_1 = \hat{Q}_1 + \hat{P}_4$ ext angle of Δ = op pint angles/ <i>Verl hoek van Δ = opp hoeke</i></p> <p>$= 32^\circ + 32^\circ$</p> <p>$\therefore \hat{M}_1 = 64^\circ$</p>	<p>✓ S</p> <p>✓ answer</p> <p>(2)</p>
9.3.4	<p>$\hat{S} = 180^\circ - 122^\circ$ sum of angles of $\Delta = 180^\circ$/ <i>Som van hoeke van $\Delta = 180^\circ$</i></p> <p>$\therefore \hat{S} = 58^\circ$</p> <p>But/<i>Maar</i> $\hat{S} = \hat{R} = 58^\circ$ angle sub by same chord/ <i>hoek verv by dieselfde koord</i></p>	<p>✓ ✓ S/R</p> <p>✓ ✓ S/R</p> <p>(4)</p>
		[19]

QUESTION/VRAAG 10

10.1	In the same proportion/ <i>In dieselfde verhouding</i>	✓ same proportion (1)
10.2	<p>Const: Join KZ & LY & draw h_1 from $K \perp XL$ & h_2/ <i>Konst: Verbind KZ & LY & skets h_1 van $K \perp XL$ & h_2</i></p> <p>Proof/Bewys</p> $\frac{\text{Area/Oppvl } \Delta XKL}{\text{Area/Oppvl } \Delta LYK} = \frac{\frac{1}{2}XK \times h_1}{\frac{1}{2}KY \times h_2} = \frac{XK}{KY}$ $\frac{\text{Area/Oppvl } \Delta XKL}{\text{Area/Oppvl } \Delta KLZ} = \frac{\frac{1}{2}XL \times h_2}{\frac{1}{2}LZ \times h_2} = \frac{XL}{LZ}$ <p>Area of ΔXKL = Area ΔXKL common/ <i>Oppvl van ΔXKL = Oppvl ΔXKL algemeen</i></p> <p>But Area ΔLKY = Area ΔKLZ same base & height; $LK \parallel YZ$/ <i>Maar Oppvl ΔLKY = Oppvl ΔKLZ selfde basis & hoogte; $LK \parallel YZ$</i></p> $\frac{\text{Area/Oppvl } \Delta XKL}{\text{Area/Oppvl } \Delta LYK} = \frac{\text{Area/Oppvl } \Delta XKL}{\text{Area/Oppvl } \Delta KLZ}$ $\therefore \frac{XK}{KY} = \frac{XL}{LZ}$	<p>✓ const</p> $\checkmark \frac{\text{Area } \Delta XKL}{\text{Area } \Delta LYK} = \frac{XK}{KY}$ $\checkmark \frac{\text{Area } \Delta XKL}{\text{Area } \Delta KLZ} = \frac{XL}{LZ}$ <p>✓ S ✓ R</p> <p>✓ S</p> <p>(6)</p>

10.3.1	$\frac{ED}{DC} = \frac{AT}{AC} = \frac{2}{3}$ <p style="text-align: center;">line // to one side of \triangle/ lyn // aan die eenkant van \triangle</p> $DE = \frac{2}{3} \times 9$ <p style="text-align: center;">= 6 and/en BD = 6 given/gegee</p> $\therefore D \text{ is the midpoint of BE/is the middelpunt van BE}$	✓ ✓ S/R ✓ answer (3)
10.3.2	$BF = FT$ <p style="text-align: center;">conv of midpoint theorem</p> $FD = \frac{1}{2}TE$ <p style="text-align: center;">midpoint theorem/middelpunt bewys</p> $\therefore TE = 4 \text{ cm}$	✓ S/R ✓ R ✓ $TE = 4$ (3)
10.3.3 (a)	$\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$ $= \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times b_2 \times h} \text{ same height/selfde hoogte}$ $= \frac{3y}{2y}$ $= \frac{3}{2}$	✓ ✓ S/R ✓ $\frac{3}{2}$ (3)
(b)	$\frac{\frac{1}{2} \times TC \times EC \times \sin C}{\frac{1}{2} \times AC \times BC \times \sin C}$ $= \frac{(x)(y)}{(3x)(5y)}$ $= \frac{1}{15}$	✓ ✓ $\frac{\frac{1}{2} \times TC \times EC \times \sin C}{\frac{1}{2} \times AC \times BC \times \sin C}$ ✓ $\frac{1}{15}$ (3)
		[19]

QUESTION/VRAAG 11

<p>11.1</p>	<p>In $\triangle TPS$ and $\triangle QSR$</p> $\frac{PS}{QS} = \frac{1,5}{4} = \frac{3}{8}$ $\frac{TP}{SR} = \frac{4,5}{12} = \frac{3}{8}$ $\frac{TS}{QR} = \frac{3,6}{9,6} = \frac{3}{8}$ <p>$\therefore \triangle TPS \sim \triangle QSR$ sides of triangles in proportion <i>sye van die driehoeke is in verhouding</i></p> <p>$\therefore \hat{P} = \hat{R}$</p> <p>$\therefore TP$ is a tangent converse of tan chord theorem/ <i>is 'n raaklyn teenoorg. van tan koordstelling</i></p>	$\checkmark \frac{PS}{QS} = \frac{1,5}{4}$ $\checkmark \frac{TP}{SR} = \frac{4,5}{12}$ $\checkmark \frac{TS}{QR} = \frac{3,6}{9,6}$ <p>$\checkmark \checkmark S/R$</p> <p>$\checkmark \hat{P} = \hat{R}$</p> <p>$\checkmark R$</p> <p>(7)</p>
<p>11.2</p>	<p>$\hat{P} = \hat{QSR}$ $\triangle s \sim$</p> <p>$QS \parallel TP$ corr angles =/korr hoeke</p> $\frac{TQ}{9,6} = \frac{1,5}{12}$ <p>$\therefore TQ = 1,2$</p> <p>proportional theorem/<i>verhoudingstelling</i></p>	<p>$\checkmark S$</p> <p>$\checkmark R$</p> <p>$\checkmark \checkmark S/R$</p> <p>$\checkmark TQ = 1,2$</p> <p>(5)</p>
<p>[12]</p>		

TOTAL/TOTAAL: 150